

I started with the weighted average price of primary energy. I used the data from EIA's Annual Energy Review which had the price data and amount of energy produced each year by the various primary energy sources where $\left\langle \frac{\partial M}{\partial E} \right\rangle$ has units of \$/kJ. M

represents money.

$$\left\langle \frac{\partial M}{\partial E} \right\rangle = \sum_i w_i \left\langle \frac{\partial M}{\partial E} \right\rangle_i$$

I then did a calibration trick. As utility in economic sense is defined by a positive affine transformation. I set the marginal utility of energy to 1:

$$\left\langle \frac{\partial U}{\partial E} \right\rangle = 1$$

The $\langle \rangle$ represent a canonical average.

$$\left\langle \frac{\partial M}{\partial E} \right\rangle = \left\langle \frac{\partial M}{\partial U} \right\rangle$$

Inverting results in the marginal utility of money:

$$\left\langle \frac{\partial U}{\partial M} \right\rangle$$

This has the same physical impact and meaning as pressure.

As the marginal utility of anything is not fixed I had to provide a reference. The traditional way that this is done is through the CPI which is just a ratio of the marginal utilities of the average price of goods and services referenced to the 1914 level. I was much more formal I referenced it to how much does it cost to by the energy to raise 1kg of water 1°K. This is simply a pressure calibration of value of the dollar. Because our economy does not work in kJ one can simply multiply the price of some item j to determine the marginal utility referenced to a joule. This makes a lot of intuitive sense, especially to someone with an engineering bent.

Now when we take our item j and we want to determine its marginal utility:

$$\left\langle \frac{\partial U}{\partial N} \right\rangle_j = \left\langle \frac{\partial U}{\partial M} \right\rangle \left\langle \frac{\partial M}{\partial N} \right\rangle_j$$

Now how we get to the engineering units is that we note that a barrel of oil has a defined volume of 0.15897295 m^3 Natural gas which is similarly priced but on a ft^3 basis. Thus kJ/m^3 becomes kPa .

What gets more wild is when you say that over the span of a day the economy is adiabatic and apply perturbation theory to the economy saying that a small perturbation in price does not effect the flow of capital or the distribution of capital.

Define:

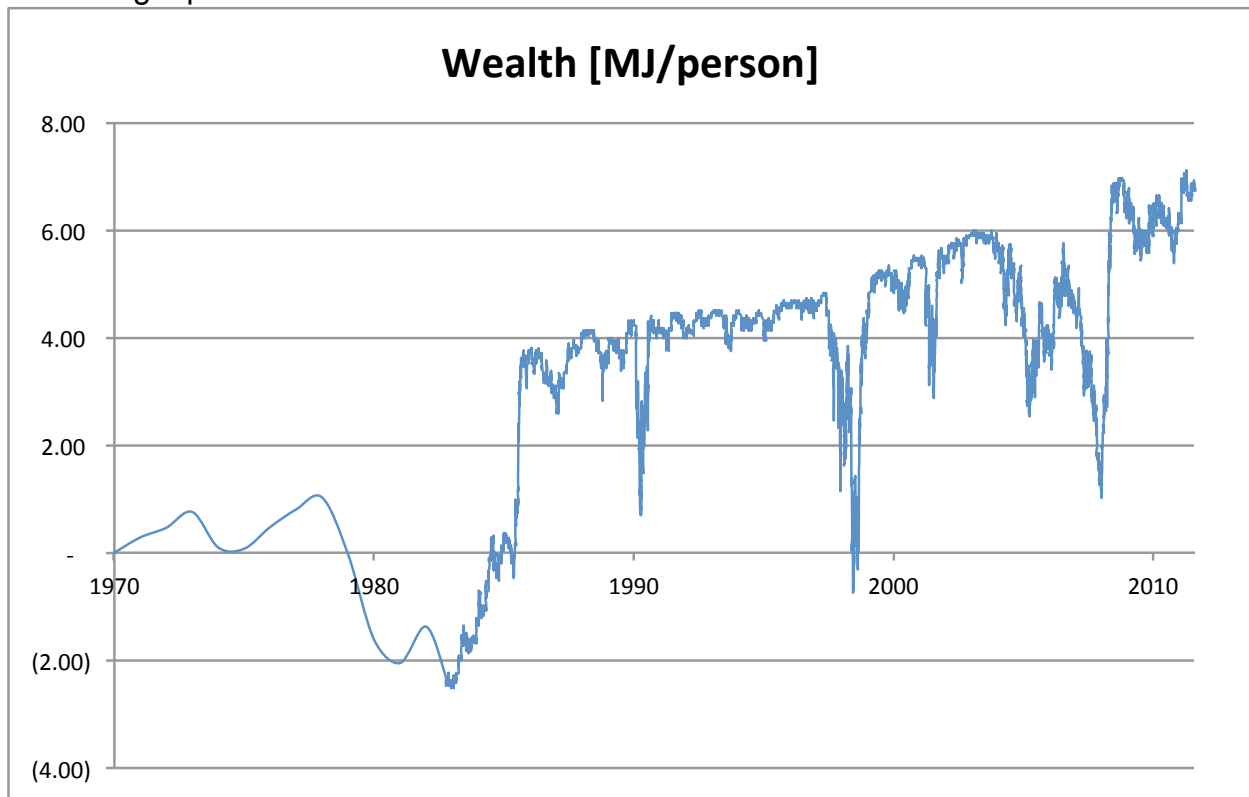
$$\lambda = \left\langle \frac{\partial U}{\partial M} \right\rangle$$

Using a Cobb-Douglas production function for a single commodity market (energy in this case), we note that it has the same functional form as the ideal gas law. Using the adiabatic relationship and the fact the hamiltonian which defines the canonical distribution is self adjoint we find:

$$\frac{\lambda_i}{\lambda_{i-1}} = \left(\frac{M_{i-1}}{M_i} \right)^\gamma$$

Doing a regression on this over the past 30-years and using M2 as the measure of monetary supply we find $\gamma = 1.162 \pm 0.018$.

We then get plots like this:



In thermodynamics, $\langle \partial U / \partial N_j \rangle$ is considered the chemical potential of the j th type of particle.

Using the same analogy, and duplicate mathematical operations we will consider the marginal utility to be the economic potential of the j th commodity. And represent the expected marginal utility as economic potential, $MU_j = \mu_j$. The total differential of a system's expected utility then becomes,

$$d\langle U \rangle = Tds + \langle \lambda \rangle dM + \sum_j \left\langle \frac{\partial U}{\partial N_j} \right\rangle dN_j$$